# Quantum Algorithm II

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# Outline

# Setting Environment

# Setting Environment

1. Qiskit vs. Cirq
2. Install Jupyter Notebook
3. IBMQ

# Setting Environment

## Quskit vs. Cirq

Accessibility to examples and documents

# Setting Environment

## Jupyter Notebook

1. Anaconda is too heavy.
2. To use LaTeX, Google colab is inappropriate.

# Setting Environment

## Jupyter Notebook

1. www.python.org - download - Add Python to Path
2. Logo key + R - powershell - type 'python'
3. type '! pip install jupyter'

# Setting Environment

## IBMQ

1. Sign in
2. Launch Composer
3. Launch Lab - Notebook - Qiskit v0.35.0

# Implementation of QFT

# Implementation of QFT

## General Information

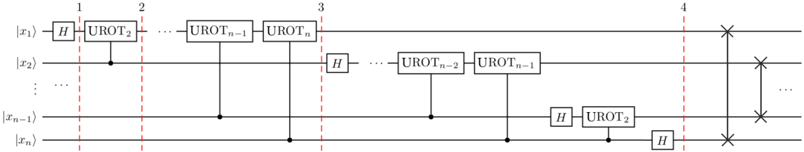
- Input : n-qubit $\boldsymbol{x} = (x_k)_{k=0}^{2^n-1} = \sum_{j=0}^{2^n-1} x_j |j\rangle$
- Output : $X = (X_k)_{k=0}^{2^n-1}$ defined by

$$X_k = \frac{1}{2^{n/2}} \sum_{j=0}^{2^n-1} x_j \exp\left(i\frac{2\pi}{N}jk\right)$$

- Complexity : width $O(n)$, depth $O(n^2)$

# Implementation of QFT

## Circuit



# Implementation of QFT

## Algorithm representation

$$|x_1 x_2 \cdots x_n\rangle \xrightarrow{H_1} \frac{1}{2^{1/2}} \left( |0\rangle + \exp\left(\frac{2\pi i}{2} x_1\right) |1\rangle \right) |x_2 \cdots x_n\rangle$$

$$\xrightarrow{\text{UROT}_2} \frac{1}{2^{1/2}} \left( |0\rangle + \exp\left(\frac{2\pi i}{2^2} x_2 + \frac{2\pi i}{2} x_1\right) |1\rangle \right) |x_2 \cdots x_n\rangle$$

$$\cdots$$

$$\xrightarrow{\text{UROT}_n} \frac{1}{2^{1/2}} \left( |0\rangle + \exp\left(\frac{2\pi i}{2^n} x\right) |1\rangle \right) |x_2 \cdots x_n\rangle .$$

$$\cdots$$

$$\longrightarrow \frac{1}{2^{n/2}} \bigotimes_{k=0}^{n-1} \left( |0\rangle + \exp\left(\frac{2\pi i}{2^{n-k}} x\right) |1\rangle \right)$$

$$\xrightarrow{\text{Rev}} \frac{1}{2^{n/2}} \bigotimes_{k=1}^{n} \left( |0\rangle + \exp\left(\frac{2\pi i}{2^k} j\right) |1\rangle \right)$$

# Implementation of QFT

## What we need

1. Circuit
2. Quantum Gate(H, UROT)
3. Debug
4. Measurement

# Implementation of QFT

See my IBMQ notebook.

# Implementation of Quantum Phase Estimation

# Implementation of QPE

## General Information

- Input : Given unitary operator $U$ with eigenvalue equation

$$U|\phi\rangle = e^{2\pi i\varphi}|\phi\rangle$$

- Output : An estimate of $\varphi \in [0, 1)$.
- Complexity : depth $O\left(n^2\right)$, width $O(n)$

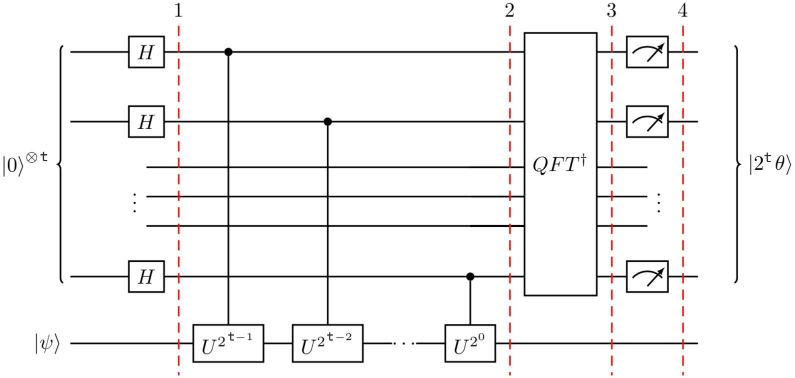
# Implementation of QPE

## Algorithm representation

$$
\begin{aligned}
|00\cdots 0\rangle|\phi\rangle &\xrightarrow{H^{\otimes n}} \frac{1}{2^{n/2}}\sum_{k=0}^{2^n-1}|k\rangle|\phi\rangle \\
&\xrightarrow{\text{CU}} \frac{1}{2^{n/2}}\sum_{k=0}^{2^n-1}|k\rangle U^k|\phi\rangle = \frac{1}{2^{n/2}}\sum_{k=0}^{2^n-1}e^{2\pi ik\varphi}|k\rangle|\phi\rangle \\
&\xrightarrow{\text{QFT}^{-1}} \frac{1}{2^n}\sum_{k,l}\exp\left(2\pi ik\left(\varphi-\frac{l}{2^n}\right)\right)|l\rangle|\phi\rangle . \\
&\xrightarrow{\text{Measure}} P\left(\text{observe nearest integer a to } 2^n\varphi\right) \geq \frac{4}{\pi^2}
\end{aligned}
$$

# Implementation of QPE

## Circuit



# Implementation of QPE

## Application

- Input : Given unitary operator

$$P = \begin{pmatrix} 1 & 0 \\ 0 & e^{i} \end{pmatrix}$$

with eigenvalue equation

$$U|1\rangle = e^{2\pi i(1/\pi)}|1\rangle$$

- Output : An estimate of $1/\pi \in [0, 1)$.

# Implementation of QPE

See my IBMQ notebook.

# References

The Jupyter Book Community (2021).
*Quantum Fourier Transform, Qiskit*.
https://qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html

The Jupyter Book Community (2021).
*Quantum Phase Estimation, Qiskit*.
https://qiskit.org/textbook/ch-algorithms/quantum-phase-estimation.html