Chap 5: Multifingered Hand Kinematics.

1 Grasping: Given models of object, set of fingers,

contact between fingers and object

grasping is appropriate setting of forces acting at each contact.

Desirable property: (1) Force-closure

Fingers can resist arbitrary external forces. ← static

(2) Manipulability.

Fingers can accommodate arbitrary object motions. ← dynamic.

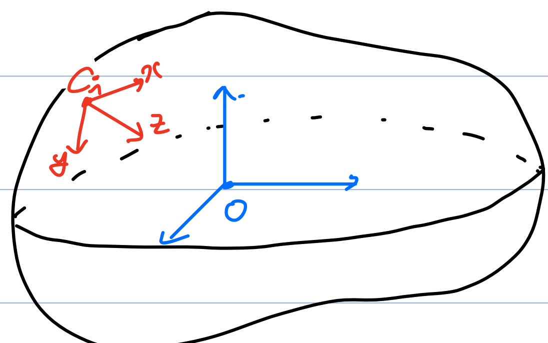
2 Grasp statics.

Here, we assume that

1) Every contact between fingers and objects are idealized as a point with a fixed location.

2) We ignore the kinematics of fingers, we consider only the transmission of a force.

Contact models and contact frame.



Object frame lies on its center of mass.

Contact coordinate frame:

the z-axis is in direction of the inward normal.

We describe the contact location as $g_{c_i} = (p_{c_i}, R_{c_i}) \in SE(3)$

The wrench F_{c_i} applied to c_i is represented as follows, depending on the model we choose.

① Frictionless point contact

$$F_{c_i} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_{c_i}, \quad f_{c_i} \geq 0$$

② Point contact with friction.

$$F_{c_i} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \\ & & \\ & & \\ & & \end{bmatrix} f_{c_i}. \quad f_{c_i} \in \underbrace{FC_{c_i}}_{\text{friction cone of } C_i} \quad FC_{c_i} := \left\{ f \in \mathbb{R}^3 \,\middle|\, \sqrt{f_1^2 + f_2^2} \leq \mu f_3,\ f_3 \geq 0 \right\}$$

③ Soft finger

$$F_{c_i} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \\ & & & 0 \\ & & & 1 \end{bmatrix} f_{c_i}, \quad f_{c_i} \in FC_{c_i}, \quad FC_{c_i} := \left\{ f \in \mathbb{R}^4 \,\middle|\, \begin{matrix} \sqrt{f_1^2 + f_2^2} \leq \mu f_3,\ f_3 \geq 0, \\ |f_4| \leq \gamma f_3 \end{matrix} \right\}$$

In short, we write $F_{c_i} = \underbrace{B_{c_i}}_{\text{Wrench Basis } \in \mathbb{R}^{p \times m_i}} f_{c_i}$ with $f_{c_i} \in FC_{c_i}$

$p = 3$ for planar grasp

$p = 6$ for spatial grasp.

Note: Basic property of cone:

① FC_{c_i} is a closed subset of $\mathbb{R}^{m_i}$ with non-empty interior.

② $f_1, f_2 \in FC_{c_i} \Rightarrow \alpha f_1 + \beta f_2 \in FC_{c_i}$ for $\alpha, \beta > 0$.

The grasp map.

As we did in Chap 3, we may represent the wrench exerted by a contact in object coordinate:
$$F_o = Ad^T_{g^{-1}_{c_i}} F_{c_i} = \underbrace{\begin{bmatrix} R_{c_i} & 0 \\ \hat{p}_{c_i} R_{c_i} & R_{c_i} \end{bmatrix}}_{\text{Wrench transformation matrix.}} F_{c_i} = \underbrace{\begin{bmatrix} R_{c_i} & 0 \\ \hat{p}_{c_i} R_{c_i} & R_{c_i} \end{bmatrix} B_{c_i}}_{=:\, G_i} f_{c_i}$$

Then with k contact, we may write
$$F_o = G_1 f_{c_1} + \cdots G_k f_{c_k} = \begin{bmatrix} G_1 & \cdots & G_k \end{bmatrix} \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix}$$

We define $G := \begin{bmatrix} G_1 & \cdots & G_k \end{bmatrix} = \begin{bmatrix} Ad^T_{g^{-1}_{c_1}} B_{c_1} & Ad^T_{g^{-1}_{c_2}} B_{c_2} & \cdots & Ad^T_{g^{-1}_{c_k}} B_{c_k} \end{bmatrix}$

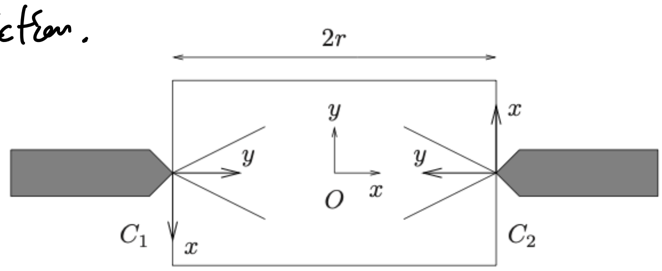
With this, we may write $F_o = G f_c$, $f_c \in FC$

where $f_c = (f_{c_1}, \cdots f_{c_k}) \in \mathbb{R}^m$, $FC = FC_1 \times \cdots \times FC_k \subset \mathbb{R}^m$, $m = \sum m_i$.

Example 5.1. Grasp map for frictionless point contacts.

In this setting, we have $G_i = \begin{bmatrix} R_{c_i} & \\ \hat{p}_{c_i} R_{c_i} & R_{c_i} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{c_i} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ \hat{p}_{c_i} R_{c_i} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} n_{c_i} \\ p_{c_i} \times n_{c_i} \end{bmatrix}$

Example 5.3 Planar grasp of a rectangle, with friction.



$R_{c_1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $R_{c_2} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $p_{c_1} = \begin{bmatrix} -r \\ 0 \end{bmatrix}$, $p_{c_2} = \begin{bmatrix} r \\ 0 \end{bmatrix}$

$B_{c_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, $FC_i = \{ f \in \mathbb{R}^2 \mid |f_1| \le \mu f_2,\ f_2 \ge 0 \}$

$$\Rightarrow G = \begin{bmatrix} Ad^T_{g^{-1}_{c_1}} B_{c_1} & Ad^T_{g^{-1}_{c_2}} B_{c_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ r & 0 & r & 0 \end{bmatrix}.$$

3 Force Closure.

Def. A grasp is force-closure if given $F_e \in \mathbb{R}^p$, $\exists f_c \in FC$ s.t. $F_e = -Gf_c$.

Prop. A grasp is force-closure iff $G(FC) = \mathbb{R}^p$.

Def. If $f_N \in \operatorname{Ker} G \cap FC$, then f_N is internal force

If $f_N \in \operatorname{Ker} G \cap \operatorname{int}(FC)$, then f_N is strictly internal force.

Prop. A grasp is force-closure iff G is surjective and $\exists f_N$: strictly internal force.

($\Rightarrow$) Choose $f_1 \in \operatorname{int}(FC)$ s.t. $F_0 = Gf_1 \neq 0$, choose $f_2 \in FC$ s.t. $Gf_2 = -F_0$

$\Rightarrow f_N = f_1 + f_2$ satisfies $Gf_N = 0$, $f_N \in \operatorname{int}(FC)$

($\Leftarrow$) Choose $F_0 \in \mathbb{R}^p$, and let $f_c' \in \mathbb{R}^m$ so that $F_0 = Gf_c'$. Now for strictly internal f_N,

Note that $\lim\limits_{\alpha \to \infty} \dfrac{f_c' + \alpha f_N}{\alpha} = f_N \in \operatorname{int}(FC)$

$\Rightarrow \exists \alpha > 0$ s.t. $f_c' + \alpha f_N \in \operatorname{int}(FC)$.

then, $f_c := f_c' + \alpha f_N$ satisfies $Gf_c = Gf_c' = F_0$. ∎

Constructive force-closure condition.

In frictionless point contact model, grasp map has the form

$$G = \begin{bmatrix} n_{c_1} & \cdots & n_{c_k} \\ p_{c_1} \times n_{c_1} & & p_{c_k} \times n_{c_k} \end{bmatrix}, \quad FC = \{ f \in \mathbb{R}^k,\ f_i \geq 0 \}$$

$\Rightarrow G(FC) = \mathbb{R}^6$ iff positive linear combinations of the columns of G spans $\mathbb{R}^6$.

Prop

Proposition 5.3. Convexity conditions for force-closure grasps *Consider a fixed contact grasp which contains only frictionless point contacts. Let $G \in \mathbb{R}^{p \times m}$ be the associated grasp matrix and let $\{G_i\}$ denote the columns of G. The following statements are equivalent:*

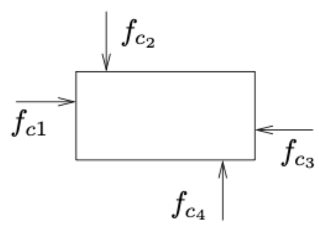
1. *The grasp is force-closure.*
2. *The columns of G positively span $\mathbb{R}^p$.*
3. *The convex hull of $\{G_i\}$ contains a neighborhood of the origin.*
4. *There does not exist a vector $v \in \mathbb{R}^p$, $v \neq 0$, such that for $i = 1, \ldots, m$, $v \cdot G_i \geq 0$.*

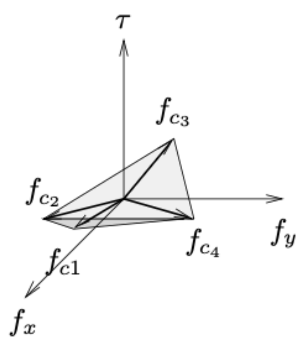
Note: 3) is equivalent to 2) since $co(\{G_1, \cdots G_m\}) = \{ \sum \alpha_i G_i \mid \sum \alpha_i = 1, \alpha_i \geq 0 \}$

4) is equivalent to 3) since

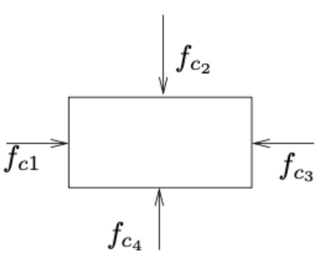
such v defines the separating hyperplane through origin.

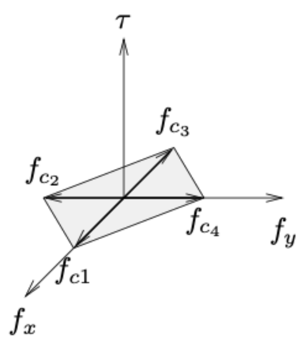
Example 5.4. We consider two different grasp on rectangle.





$$\Rightarrow G = \begin{bmatrix} 1 & & -1 & \\ & -1 & & 1 \\ -a & b & -a & b \end{bmatrix}$$





$$\Rightarrow G = \begin{bmatrix} 1 & & -1 & \\ & -1 & & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

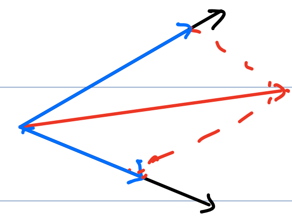
Here, 4) also fails.

Example 5.5. Grasp with friction.

In planar case, all contact force which lies in friction cone

is a positive liner combinations of the forces that describe the edge of cone.

This is equivalent to defining contact map

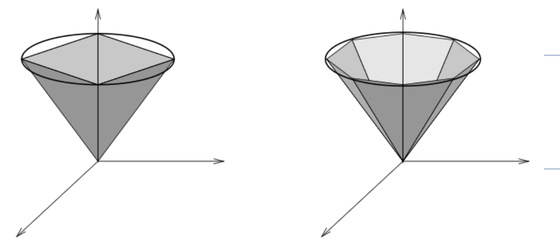


$G_i' = \begin{bmatrix} 1 & 1 \\ \mu & -\mu \\ \mu r & -\mu r \end{bmatrix}$, $FC_{c_i}' = \{ f \in \mathbb{R}^2 \mid f_1, f_2 \geq 0 \}$

Hence we may use the same idea of proposition..

In Spatial Case, one can approximate the friction cone

by the positive span of finite set of vectors.



4 Grasp Planning,

So far, we have discussed whether a grasp is force-closure with given contact point.

In this section, we consider following two question.

① Given contact model, how many fingers are necessary to grasp any object?

② Under the condition above, how can we determine the contact point?

Given an object $O \subset \mathbb{R}^3$ (or $O \subset \mathbb{R}^2$ in planar). Let $\Sigma = \partial O$.

Assume Σ is connected, piecewise smooth.

and let $\Lambda(\Sigma)$ be set of all wrenches that can be applied to O using frictionless point contact.

$$\Lambda(\Sigma) = \left\{ \begin{bmatrix} n_{c_i} \\ p_{c_i} \times n_{c_i} \end{bmatrix} \middle| \, c_i \in \Sigma \right\}$$

<u>Def</u> The surface Σ is exceptional if $\mathrm{co}(\Lambda(\Sigma))$ does not contain a nbd of origin.

→ examples of exceptional surfaces are sphere in $\mathbb{R}^3$, or circle in $\mathbb{R}^2$.

For non-exceptional surfaces, we have upper and lower bound for the # of contacts required for a force-closure grasp.

Theorem 5.4 (Caratheodory). *If a set $X = \{v_1, \ldots, v_k\}$ positively spans $\mathbb{R}^p$, then $k \geq p+1$.*

Theorem 5.5 (Steinitz). *If $S \subset \mathbb{R}^p$ and $q \in \mathrm{int}(\mathrm{co}\, S)$, then there exists $X = \{v_1, \ldots, v_k\} \subset S$ such that $q \in \mathrm{int}(\mathrm{co}\, X)$ and $k \leq 2p$.*

These are the classical thm in convex analysis.

Then we have $p=3$ (planar) $\Rightarrow 4 \leq k \leq 6$

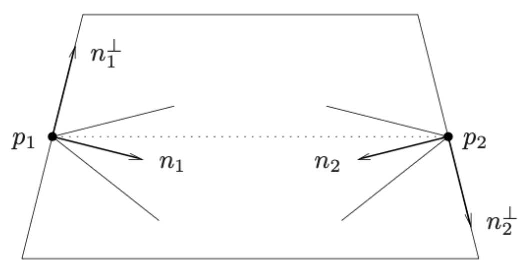
$p=6$ (spatial) $\Rightarrow 7 \leq k \leq 12$.

For the planar case, the lower bound is achievable for all non-exceptional surfaces.

Constructing force-closure grasps.

In planar grasp, we introduce one algorithm that constructs force-closure grasps for the case of two point contacts with friction.

Theorem 5.6. Planar antipodal grasps [82]
A planar grasp with two point contacts with friction is force-closure if and only if the line connecting the contact point lies inside both friction cones.



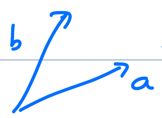
The algorithm is based on the fact that,

the condition of the thm can be written as a set of linear inequality constraint.

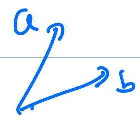
$A: (n_1 - \mu n_1^{\perp}) \otimes (p_2 - p_1) > 0$ $\quad$ $B: (n_1 + \mu n_1^{\perp}) \otimes (p_2 - p_1) < 0$

$C: (n_2 - \mu n_2^{\perp}) \otimes (p_1 - p_2) > 0,$ $\quad$ $D: (n_2 + \mu n_2^{\perp}) \otimes (p_1 - p_2) < 0.$

$a \otimes b = \det [a \ b]$
$= a_1 b_2 - a_2 b_1.$



$\Rightarrow a \otimes b > 0$



$\Rightarrow a \otimes b < 0$

This has also extension to spatial case using the following thm.

Theorem 5.7. Spatial antipodal grasps [82]
A spatial grasp with two soft-finger contacts is force-closure if and only if the line connecting the contact point lies inside both friction cones.

In this case, the constraints are now nonlinear functions of the contact location.