Ch 9. Control of Multi-Input Physical Systems.

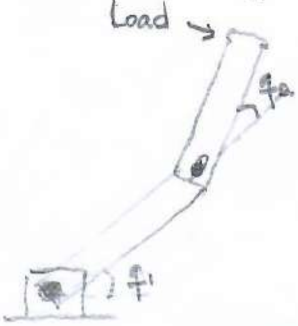
- How to design a controller in a physical stance rather than mathematical.

9.1 9.2 | 9.3 9.4 |

robot manipulator — general classes of systems

9.1. Position & Trajectory Control.

9.2 Adaptive Trajectory Control



$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau, \quad q = [q_1\ q_2]^T$$

where

- $H(q) \in \mathbb{R}^{2\times 2}$: inertia matrix (sym & pos. def.)
- $C(q,\dot{q})\dot{q}$: Centripetal & Coriolis torque
- $g(q)$: gravitational torque
- τ : control input to each joint (control law)

Q. How to construct τ to achieve $q \to q_d$.

- q_d : const. : position control
- $q_d = q_d(t)$: trajectory control

· 2 Important Notes before we start.

① $E_k = \frac{1}{2}\dot{q}H(q)\dot{q}$ $\left(\approx \frac{1}{2}mv^2\right)$

② $H(q)$ is uniformly pos. definite i.e. $H(q) \geq \alpha I_2$ for some $\alpha > 0$

pf) If not, $\exists \{q_n\} \subseteq \mathbb{R}^2$ s.t. $\lambda_{min}(H(q_n)) \to 0$ as $n \to \infty$.) workspace is closed (compact)

$\Rightarrow \exists \{q_{n'}\} \subset \{q_n\}$ s.t. $q_{n'} \to q_\infty$ (B-W thm)

$\Rightarrow H(q_\infty)$ has zero eigenvalue

$\Rightarrow \exists \dot{q} \neq 0$ s.t. $E_k = \frac{1}{2}\dot{q}H(q_\infty)\dot{q} = 0$ ↯

9.1.1. Positional Control (q_d : const. vector)

We use a P.D controller for each joint independently

i.e. $\tau_j = -k_{P_j}\tilde{q}_j - k_{D_j}\dot{q}_j$ where $\tilde{q}_j = q_j - q_{d_j}$ $(j = 1, 2,\ k_{P_j}, k_{D_j} > 0)$

$\Rightarrow \tau = -K_P\tilde{q} - K_D\dot{q}$

manip. on horizontal plane.

Transferring the problem to Hamiltonian form, assuming $g(q) = 0$,

$$\frac{1}{2}\frac{d}{dt}(\dot{q}^T H\dot{q}) = \frac{1}{2}\ddot{q}^T H\dot{q} + \frac{1}{2}\dot{q}^T(\dot{H}\dot{q} + H\ddot{q})$$

$\dot{q}^T\dot{H}\dot{q} \approx 0$ often neglected in control context.

$$\approx \frac{1}{2}\ddot{q}^T H\dot{q} + \frac{1}{2}\dot{q}^T H\ddot{q}$$

$$= \dot{q}^T H\ddot{q}$$

$$= \dot{q}^T(\tau - C\dot{q} - \cancel{g}^{\,0})$$

$\dot{q}C\dot{q}$ neglected. (related with $\dot{H}$)

$$= \dot{q}^T\tau$$

$$= \dot{q}^T(-K_P\tilde{q} - K_D\dot{q})$$

pot. E ($\approx \frac{1}{2}kx^2$).

mech. Energy $V = \frac{1}{2}[\dot{q}^T H\dot{q} + \tilde{q}^T K_P\tilde{q}]$

$$\Rightarrow \dot{V} = \dot{q}^T(-K_P\tilde{q} - K_D\dot{q}) + \frac{1}{2}\frac{d}{dt}(\tilde{q}^T K_P\tilde{q})$$

$\dot{\tilde{q}} = \frac{d}{dt}(q - q_d) = \dot{q}$

$$= \dot{q}^T(-K_P\tilde{q} - K_D\dot{q}) + \tilde{q}^T K_P\dot{q}$$

$\dot{q}^T K_P\tilde{q} = \tilde{q}^T K_P\dot{q}$

$$= -\dot{q}^T K_D\dot{q} \le 0$$

By Barbalat's lemma ($\dot{V}$ is bounded $\Rightarrow V$: UC), $\dot{V} \to 0$ as $t \to \infty$.

We should also check system doesn't get stucked. (goal : $q \to q_d$)

i.e. $\dot{V} = 0$ only when $\tilde{q} = 0$

pf) $\dot{V} = 0 \Leftrightarrow \dot{q} = 0$

$$\Rightarrow \ddot{q} = H^{-1}\tau = H^{-1}(-K_P\tilde{q} - \cancel{K_D\dot{q}}^{\,0})$$

$$= -H^{-1}K_P\tilde{q}$$

$\Rightarrow \dot{q}$ is identically 0 only if $\tilde{q} = 0$.

9.1.2 Trajectory Control ($q_d = q_d(t)$)

· Feedback Linearization

$$\tau = H v + C\dot{q} + g \quad \text{where} \quad v = \ddot{q}_d - 2\lambda \dot{\tilde{q}} - \lambda^2 \tilde{q} \quad (\lambda > 0)$$

$\Rightarrow$ $H\ddot{q} + C\dot{q} + g = \tau$ turns into

$$\ddot{q} = v = \ddot{q}_d - 2\lambda \dot{\tilde{q}} - \lambda^2 \tilde{q}$$

$\Rightarrow$ $\ddot{\tilde{q}} + 2\lambda \dot{\tilde{q}} + \lambda^2 \tilde{q} = 0$. i.e $\tilde{q}$ exponentially converges to 0.

· Robust Control.

Aim to handle uncertainty & disturbances in the system.
We derive a robust traj. controller using sliding control (chap 7)

sliding condition for multi-input case is

$$\frac{1}{2}\frac{d}{dt}(s^T s) \le -\eta (s^T s)^{1/2} \quad (\eta > 0) \quad \text{where} \quad s = \underbrace{\dot{\tilde{q}}}_{\text{velocity error}} + \underbrace{\Lambda}_{\text{sym. pos. def.}} \tilde{q}$$

$$= \dot{q} - \dot{q}_r \quad (\dot{q}_r = \dot{q}_d - \Lambda \tilde{q})$$

Note

① Assuming bdd initial cond. s : bdd $\Leftrightarrow$ $\tilde{q}$ & $\dot{\tilde{q}}$ bdd.

② $s \to 0 \Rightarrow \tilde{q}$ & $\dot{\tilde{q}} \to 0$

③ Look at Ham. Form again with $g \neq 0$

$$\dot{q}^T(\tau - g) = \frac{1}{2}\frac{d}{dt}[\dot{q}^T H \dot{q}]$$

$$\Rightarrow \dot{q}^T(H\ddot{q} + C\dot{q} + \cancel{g} - \cancel{g}) = \dot{q}^T H \ddot{q} + \frac{1}{2}\dot{q}^T \dot{H} \dot{q}$$

$$\Rightarrow \dot{q}^T(\dot{H} - 2C)\dot{q} = 0.$$

$\Rightarrow$ $\dot{H} - 2C$ is skew-symmetric

not quite obvious $\Rightarrow$ Accept.
but proof is too long (p.400).

$$\Rightarrow (\dot{H} - 2C)^T = -(\dot{H} - 2C)$$

$$\Rightarrow \dot{H} = C + C^T$$

9.2 Adaptive Robot Trajectory Control.

· We deal with the situation when the manipulator's mass properties are unknown. But want $q \to q_d$

a : unknown param. of mass properties (const. vector).

$\tilde{a} = \hat{a} - a$

$V(t) = \frac{1}{2}[s^T H s + \tilde{a}^T \Gamma^{-1} \tilde{a}]$ where Γ is a sym. pos. def. matrix

$\Rightarrow \dot{V}(t) = s^T(\tau - H\ddot{q}_r - C\dot{q}_r - g) + \dot{\hat{a}}^T \Gamma^{-1} \tilde{a}$

Since H, C, g all linearly depends on a, we can construct Y s.t.

$H\ddot{q}_r + C\dot{q}_r + g = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\, a$

Now, take $\tau = Y\hat{a} - K_D s$ using P.D controller,

$\Rightarrow \dot{V} = s^T(Y\hat{a} - K_D s - Ya) + \dot{\hat{a}}^T \Gamma^{-1} \tilde{a}$

$= s^T Y \tilde{a} - s^T K_D s + \dot{\hat{a}}^T \Gamma^{-1} \tilde{a}.$

update $\hat{a}$ by $\dot{\hat{a}} = -\Gamma Y^T s$

$\Rightarrow \dot{V} = -s^T K_D s \le 0$: global stability & convergence of $\tilde{q}$ to 0

Dealing with prob.

Define $V(t) = \frac{1}{2} s^T H s$

$\Rightarrow \dot{V}(t) = s^T H \dot{s} + \frac{1}{2} s^T \dot{H} s$

$= s^T H (\ddot{q} - \ddot{q}_r) + \frac{1}{2} s^T \dot{H} s \quad \downarrow s = \dot{q} - \dot{q}_r$

$\downarrow H\ddot{q} = \tau - C\dot{q} - g$

$= s^T (\tau - C\dot{q} - g - H\ddot{q}_r) + \frac{1}{2} s^T \dot{H} s$

$= s^T \{ \tau - C(s + \dot{q}_r) - g - H\ddot{q}_r \} + \frac{1}{2} s^T \dot{H} s$

$= s^T (\tau - H\ddot{q}_r - C\dot{q}_r - g) + \frac{1}{2} s^T (\dot{H} - 2C) s$

Choose $\tau = \hat{\tau} - K \cdot \mathrm{sgn}(s)$ (const. vector we can chose; element wise) Where $\hat{\tau} = \hat{H}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{g}$ (makes $\dot{V} = 0$ given all dynamics known)

$\Rightarrow \dot{V} = s^T [\tilde{H}\ddot{q}_r + \tilde{C}\dot{q}_r + \tilde{g}] - \sum_i k_i |s_i|$ $\quad (\tilde{H} = \hat{H} - H)$

Given bounds of modelling errors $\tilde{H}, \tilde{C}, \tilde{g}$, if we choose k_i s.t.

$k_i \geq |[\tilde{H}\ddot{q}_r + \tilde{C}\dot{q}_r + \tilde{g}]_i| + \eta_i$, then $\dot{V} \leq -\sum_i \eta_i |s_i|$

we get the desired sliding cond.

$\Rightarrow$ sliding surface $s = 0$ is reached in finite time

& $q \rightarrow q_d$ exponentially.