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9.3. Putting Physics in Control

We'll extend our earlier discussions to larger classes of systems & control problems.

9.3.1. High-Frequency Unmodeled dynamics

In control design, uncertainties are classified as

structured ⟷ unstructured

structured	unstructured
• inaccuracy of params.	• errors on system order
• additive disturbances	i.e. unmodeled dynamics
e.g. stiction(= static friction)	

Adequate modeling gives "high-frequency" unmodeled dynamics, thus we'd like to make the "bandwith" of control input lower than the frequencies of the unmodeled dynamics in order to distinguish between the two types of uncertainties.

Feedback: reduces the effects of structured uncertainties
↳ now limited in terms of freq.

However, if the effective limit of the bandwith is caused by a passive element of the actual system dynamics (e.g. structural resonant modes, aero-elastic effects, structural flexibility), we may improve our performance beyone the classic bandwith limitations. In other words, components of the control law, when combined with the passive element acts like a new passive system, can be tuned separately from the controller.

⇒ allows the rest of the control to be tuned less conservatively.

9.3.2. Conservative and Dissipative Dynamics

9.1 : conservative physical system

Dissipative terms (e.g. friction) represent additive inputs, but it is potentially more effective to exploit known dissipative characters rather then to treat them as simple perturbations.

E.g. Viscous friction torque vector is known to be $D\dot{q}$ $(D>0)$, this only introduces extra damping.

$$\tau = Y\hat{a} - K_D s \xrightarrow{D\dot{q} = Y_D(\dot{q})a_D} \begin{cases} \tau = Y\hat{a} + Y_D(\dot{q})\hat{a}_D - K_D s \\ \Rightarrow \dot{\hat{a}}_D = -\Gamma_D Y_D^T(\dot{q}) s \\ \Rightarrow \dot{V} = -s^T K_D s \end{cases}$$

$$\tau = Y\hat{a} - K_D s \xrightarrow[D\dot{q} = D\dot{q}_r + Ds]{} \begin{cases} \tau = Y\hat{a} + Y_D(\dot{q}_r)\hat{a}_D - K_D s \\ \Rightarrow \dot{V} = -s^T(K_D + D)s \end{cases}$$

(Recall $Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) = H(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + g(q) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)a$

$\hat{a}$: param estimation error

$s = \dot{\tilde{q}} + \Lambda\tilde{q}$

$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}$

↳ desired

$V = \frac{1}{2}(s^T H s + \tilde{a}^T \Gamma^{-1} \tilde{a})$

The second approaches uses $\dot{q}_d$ rather than $\dot{q}$ and tuning is more effective.

Coulomb friction ($D_c \, \mathrm{sgn}(\dot{q})$) : also adds a negative semi definite term when using $D_c \, \mathrm{sgn}(\dot{q}_r)$, instead of $D_c \, \mathrm{sgn}(\dot{q})$. $(-s^T D_c(\mathrm{sgn}(\dot{q}_r) - \mathrm{sgn}(\dot{q}))$ has same sign as s)

9.3.3. Robotics as a Metaphor

$$\frac{d}{dt}[\text{Stored Energy}] = \underbrace{[\text{External Power Input}]}_{y^t u} + [\text{Internal Power Generation}]$$

$y^t u$ (u : input vector, y : output vector)

WMA u can be used for control purposes.

Note. The system need not be dissipative or passive as long as the internal power generation can be compensated for by the external power input.

Our goal is to design a feedback controller that translates the energy conservation eqn in terms of a mapping between parametric uncertainty and $s = y - y_r$

E.g. Passivity Interpretation of Adaptive Manipulator Control

Conservation of energy : $\frac{1}{2}\frac{d}{dt}(\dot{q}^t H \dot{q}) = \dot{q}^t(\tau - g)$

$\tau - g \mapsto \dot{q}$ is passive ($g = 0 \Rightarrow$ refer to 4.7)

Control law: $\tau = Y\hat{a} - K_D s$ $\quad (s = \dot{q} - \dot{q}_r)$, then

$$\frac{1}{2}\frac{d}{dt}(s^t H s) = s^t Y\tilde{a} - s^t K_D s$$

, and with the adaptation law $\dot{\hat{a}} = -\Gamma Y^T s$,

$$\frac{1}{2}\frac{d}{dt}(\tilde{a}^t \Gamma^{-1} \tilde{a}) = -s^T Y \tilde{a}$$

$\tau - g \mapsto \dot{q}$ to

$s \mapsto -Y\tilde{a}$ (closed)

$\Rightarrow -Y^t s \mapsto \tilde{a}$, $s \mapsto -Y\tilde{a}$ is passive.

We therefore have the diagram

$$\begin{array}{c} H\dot{s} + (C + K_D)s = Y\tilde{a} \\ \uparrow \qquad\qquad \downarrow \\ \dot{\hat{a}} = -\Gamma Y^T s \end{array}$$

9.4 Spacecraft Control

Weightlessness $\Rightarrow$ expected to easily handle objects of masses and volumes larger than the spacecraft

But nature - large dynamic uncertainties $\Rightarrow$ requires effective adaptive capabilities.

9.4.1. The Spacecraft Model

Assume the spacecraft is driven by 3 orthogonal w.r.t spacecraft frame (S.F) reaction wheels (gas jets in practice for translational motion), and is rigid.

- Dynamic equation.

Take any inertial frame (I.F.).

ω: angular velocity vector of the spacecraft (w.r.t S.F.)

H (inertia matrix) > 0

$H = H^0 - H^A$

H^0 : total central inertia matrix (S.F)

H^A : matrix of axial wheels' inertias (S.F)

p: total spacecraft angular momentum (S.F)

$$\Rightarrow \underline{H\dot{\omega} = p \times \omega + \tau}$$

$[v\times]$: Matrix of $v \times (\cdot)$

- Kinematic equation

Gibbs vector: $\xi = \tan(\frac{\rho}{2}) e$ (by a quaternion parametrization) ☆

ξ: result of a rotation of ρ about an axis e. (I.F.)

$$\Rightarrow \dot{\xi} = Z(\xi)\omega \quad (Z(\xi) = \tfrac{1}{2}(I_3 + \xi\xi^t + \xi\times))$$

$p = C(\xi)\, p^I$ (coordinate transformation), where

$$C(\xi) = 2(1+\xi^t\xi)^{-1}(I_3 + \xi\xi^t - \xi\times) - I$$

$$(C^{-1}(\xi) = C^t(\xi) = C(-\xi))$$

Euler angle representation

φ, θ, ψ : roll, pitch, yaw (x, y, z axis) $\quad (-\frac{\pi}{2} < \theta < \frac{\pi}{2})$

$r = (\varphi, \theta, \psi)^t \Rightarrow \dot{r} = M(r)\,\omega$, where

$$M(r) = \begin{pmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{pmatrix}$$

, or $p = R(r)\,p^I$, where

$$R(r) = \begin{pmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\psi & \cos\psi\sin\theta & \cos\psi\cos\varphi \end{pmatrix}$$

For either representation, we have $\dot{x} = J(x)\,\omega$ $\quad (J = M \text{ or } Z)$

9.4.2 Attitude Control

$\tau = J^T F$, $H^*(x) = J^{-t} H(x) J^{-1}$, $C^*(x, \dot{x}) = -J^{-t} H J^{-1} \dot{J} J^{-1} - J^{-t}(p\times) J^{-1}$ gives

$$H^*(x)\ddot{x} + C^*(x, \dot{x})\dot{x} = F$$

. $(J^{-1}_{(x)} = 2(1 + x^t x)^{-1}[I_3 - x\times]$

✓ $\dot{H}^* - 2C^* = \dot{J}^{-t} H J^{-1} - J^{-T} H (\dot{J}^{-1}) + 2J^{-t}[p\times]J^{-1}$ is skew-sym.

a : params by $\begin{cases} 6 \text{ independent cpts of } H \text{ (sym)} \\ 3 \text{ cpts of } p^I \end{cases}$ $\Rightarrow$ "linear" dynamics

Now similar to robot manipulators (not surprising in the sense both are trajectory controllable Hamiltonian systems)

Attitude regulation.

We try simple P.D.-like control. (x_d : const → stable position control)

$$\tau = J^t F = -J^t (K_P \tilde{x} + K_D \dot{\tilde{x}})$$

, and Lyapunov candidate $V_1(t) = \frac{1}{2}(\dot{\tilde{x}}^t H^* \dot{\tilde{x}} + \tilde{x}^t K_P \tilde{x})$

pseudo-kinetic $\quad$ pseudo-potential

$\dot V_1(t) = \dot{\tilde x}^t(F + K_p\tilde x) = -\dot x^T K_D \dot x \le 0$ (power dissipated by virtual dampers

$\dot V_1 = 0 \Rightarrow \dot x = 0 \Rightarrow \ddot x = (H^*)^{-1}K_p\tilde x = 0 \Rightarrow \tilde x = 0$ ($(H^*)^{-1}$ exists)

(Can be extended to flexible appendage-equipped spacecrafts; they have mechanically passive systems)

Adaptive attitude tracking

Sp $x_d, \dot x_d, \ddot x_d$ are all bounded. Consider the Lyapunov-like ftn

$$V = \frac{1}{2}(s^t H^* s + \tilde a^T \Gamma^{-1}\tilde a)$$

, where $s = \dot{\tilde x} + \lambda\tilde x$ ($\lambda > 0$ const). Define $\dot x_r = \dot x_d - \lambda\tilde x$, then

$$\dot V = s^t(F - H^*\ddot x_r - C^*\dot x_r) + \tilde a^T\Gamma^{-1}\dot{\tilde a}.$$

Taking the control law to be

$$F = \hat H^*(x)\ddot x_r + \hat C^*(x,\dot x)\dot x_r - K_D s \quad (K_D \text{ sym. } K_D > 0)$$

yields $\dot V = s^t(\tilde H^*\ddot x_r + \tilde C^*\dot x_r - K_D s) + \tilde a^T\Gamma^{-1}\dot{\tilde a}$. The linear parametrization of the dynamics allows us to define a known matrix $Y^*(x, \dot x, \dot x_r, \ddot x_r)$ s.t

$\tilde H^*\ddot x_r + \tilde C^*(x,\dot x)\dot x_r = Y^*(x,\dot x,\dot x_r,\ddot x_r)\tilde a \Rightarrow \dot V = -s^T K_D s + \tilde a^T(\Gamma^{-1}\dot{\tilde a} + Y^{*t}s)$

. Now choosing the adaptation law $\dot{\hat a} = -\Gamma(Y^*)^T s$ gives $\dot V \le 0$.

$\dot V \to 0 \Rightarrow s \to 0 \Rightarrow \tilde x, \dot{\tilde x} \to 0$

Note. Adding $\lambda\tilde x^t K_D \tilde x$ to V changes nothing $\Rightarrow \tilde x$ is bdded

$\Rightarrow$ singularity of the Gibbs vector representation is not reached as long as the rotation exceeds π. but it can be done piecewise. ※

The control torque would be $\tau = J^t(\hat H^*\ddot x_r + \hat C^*\dot x_r - K_D s) = J^t(Y^*\hat a - K_D s)$

Note. One minor inconvenience: While we're estimating H, the central inertia matrix as part of the control, we don't actually know the position of CM. Also. our process does not estimate the parameters; we are simply generating $\hat a$ for which $\tilde x \to 0$.

Note, If the initial angular velocity of the wheels $\Omega(0)$ is known, then

$$p^I = R(-\chi(0))(H^0\omega(0) + H^A\Omega(0)) = R(-\chi(0))H^A(\omega(0)+\Omega(0)) + R(-\chi(0))H\omega(0)$$

↳ only the H terms (therefore 6 cpts) are sufficient to express the adaptive controller.

Remark. Gas-jets would need 4 more params: mass. and the position of CM.

9.5. Summary

- Exploitation of the physical properties allows simpler control & enhanced performance.